

$$\sigma_x = 3 \text{ MPa} \quad \sigma_y = -2 \text{ MPa} \quad \tau_{xy} = -4 \text{ MPa}$$

$$\sigma_{\text{méd.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{méd.}} = \frac{3 - 2}{2}$$

$$\sigma_{\text{méd.}} = 0,5 \text{ MPa (centro)}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{3 - (-2)}{2}\right)^2 + (-4)^2}$$

$$R = 4,717 \text{ MPa}$$

$$A(3, -4)$$

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$$\alpha = \arctan\left(\frac{4}{2,5}\right) - 40^\circ$$

$$\alpha = 18^\circ$$

$$\sigma_{x'} = 0,5 + 4,717 \cdot \cos 18^\circ$$

$$\sigma_{x'} = 4,986 \text{ MPa}$$

$$\sigma_{y'} = 0,5 - 4,717 \cdot \cos 18^\circ$$

$$\sigma_{y'} = -3,986 \text{ MPa}$$

$$\tau_{x'y'} = -4,717 \cdot \sin 18^\circ$$

$$\tau_{x'y'} = -1,457 \text{ MPa}$$

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$$\textcircled{2} \text{ a) } \sigma_x = 350 \text{ MPa}$$

$$\sigma_y = -200 \text{ MPa}$$

$$\tau_{xy} = 500 \text{ MPa}$$

$$\sigma_{\text{méd.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{méd.}} = \frac{350 - 200}{2}$$

$$\sigma_{\text{méd.}} = 75 \text{ MPa (centro)}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{350 - (-200)}{2}\right)^2 + (500)^2}$$

$$R = 570,64 \text{ MPa}$$

$$\sigma_1 = 75 + 570,64$$

$$\sigma_1 = 645,64 \text{ MPa}$$

$$\sigma_2 = 75 - 570,64$$

$$\sigma_2 = -495,6 \text{ MPa}$$

$$\tan(2\theta_p) = \frac{500}{275}$$

$$\tan(2\theta_p) = 1,8182$$

$$\theta_{p1} = 30,6^\circ \text{ (anti-horízontal)}$$

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$$\text{b) } \sigma_{\text{máx. plana}} = R = 571 \text{ MPa}$$

$$\sigma_{\text{méd.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{méd.}} = \frac{350 - 200}{2}$$

$$\sigma_{\text{méd.}} = 75 \text{ MPa}$$

$$\tan(2\theta_s) = \frac{275}{500}$$

$$\theta_{s1} = 14,4^\circ \text{ (horízontal)}$$

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$$a) \sigma_x = 45 \text{ MPa}$$

$$\sigma_y = 30 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

$$\sigma_{\text{med.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{med.}} = \frac{45 + 30}{2}$$

$$\sigma_{\text{med.}} = 37,5 \text{ MPa (centro)}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{45 - 30}{2}\right)^2 + (-50)^2}$$

$$R = 50,56 \text{ MPa}$$

$$A(45, -50) \quad \times$$

$$\sigma_1 = 37,5 + 50,56$$

$$\sigma_1 = 88,06 \text{ MPa}$$

$$\sigma_2 = -50,56 + 37,5$$

$$\sigma_2 = -13,06 \text{ MPa}$$

$$\tan(2\theta_p) = \frac{50}{7,5}$$

$$\tan(2\theta_p) = 6,667$$

$$\theta_{p1} = 40,7^\circ \text{ (anti-horiz)} \quad \times \times$$

$$b) \tau_{\text{máx. plano}} = R = 50,6 \text{ MPa}$$

$$\sigma_{\text{med.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{med.}} = \frac{45 + 30}{2}$$

$$\sigma_{\text{med.}} = 37,5 \text{ MPa}$$

$$\tan(2\theta_s) = \frac{7,5}{50}$$

$$\tan(2\theta_s) = 0,15$$

$$\theta_{s1} = 4,27^\circ \text{ (horiz)} \quad \times \times \times$$

$$\textcircled{4} \quad \sum \mathcal{M}_A = 0$$

$$-500 \cdot 1,25 + 2,5 \cdot F_C = 0 \quad \times$$

$$\boxed{F_C = 250 \text{ N}}$$

$$\uparrow + \sum F_y = 0$$

$$250 - 200 \cdot 1,5 + V = 0$$

$$\boxed{V = 50 \text{ N}}$$

$$\odot + \sum \mathcal{M} = 0$$

$$M - (200 \cdot 1,5) \cdot 0,75 - 250 \cdot 1,5 = 0$$

$$\boxed{M = 150 \text{ N}\cdot\text{m}}$$

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$$\sigma_D = \frac{M_y}{I} = \frac{150 \cdot 0,025}{\frac{0,1 \cdot 0,2^3}{12}}$$

$$\boxed{\sigma_D = 56,25 \text{ kPa (T)}}$$

$$\tau_D = \frac{V Q_D}{I b}$$

$$\tau_D = \frac{50 \cdot (0,0625 \cdot 0,075 \cdot 0,1)}{\left(\frac{0,1 \cdot 0,2^3}{12}\right) \cdot (0,1)}$$

$$\boxed{\tau_D = 3,516 \text{ kPa}}$$

$$\sigma_x = 56,25 \text{ kPa}$$

$$\sigma_y = 0 \text{ kPa}$$

$$\tau_{xy} = -3,516 \text{ kPa} \quad \times \times \times$$

$$\sigma_{\text{méd}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{méd}} = \frac{56,25 + 0}{2}$$

$$\sigma_{\text{méd}} = 28,125 \text{ kPa (centro)}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{56,25 - 0}{2}\right)^2 + (-3,516)^2}$$

$$\boxed{R = 28,344 \text{ kPa}}$$

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$$\varphi = \arctan\left(\frac{3,516}{28,125}\right)$$

$$\boxed{\varphi = 7,126^\circ}$$

$$\theta = 180^\circ - 120^\circ - 7,126^\circ$$

$$\boxed{\theta = 52,874^\circ}$$

$$\sigma_{x'} = 28,125 - 28,344 \cos(52,874^\circ)$$

$$\boxed{\sigma_{x'} = 11 \text{ kPa}}$$

$$\tau_{x'y'} = -28,344 \sin(52,874^\circ)$$

$$\boxed{\tau_{x'y'} = -22,6 \text{ kPa}}$$

$$N_z = 25 \cdot \left(\frac{4}{5}\right)$$

$$\boxed{N_z = 20 \text{ kN}}$$

$$V_y = 25 \cdot \left(\frac{3}{5}\right)$$

$$\boxed{V_y = 15 \text{ kN}}$$

$$M_x = 15 \cdot 0,4$$

$$\boxed{M_x = 6 \text{ kN}\cdot\text{m}}$$

$$\sigma_A = \frac{N_z}{A} + \frac{M_x \cdot y_A}{I_x} \quad \times \times$$

$$\sigma_A = \frac{20 \times 10^3}{0,08 \cdot 0,16} + \frac{(6 \times 10^3) \cdot 0,040}{\left(\frac{0,080 \times 0,160^3}{12}\right)}$$

$$\boxed{\sigma_A = 10,35 \text{ MPa (T)}}$$

$$\tau_A = \frac{V_y \cdot Q_A}{I_x \cdot t}$$

$$\tau_A = \frac{(15 \times 10^3) \cdot (0,060 \cdot 0,080 \cdot 0,040)}{\left(\frac{0,080 \cdot 0,160^3}{12}\right) (0,08)}$$

$$\boxed{\tau_A = 1,32 \text{ MPa}}$$

$$\sigma_x = 10,35 \text{ MPa}$$

$$\sigma_y = 0 \text{ MPa}$$

$$\tau_{xy} = -1,32 \text{ MPa}$$

$$\sigma_{\text{méd.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{méd.}} = \frac{10,35 + 0}{2}$$

$$\boxed{\sigma_{\text{méd.}} = 5,175 \text{ MPa (centro)}}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{10,35 - 0}{2}\right)^2 + (-1,32)^2}$$

$$\boxed{R = 5,34 \text{ MPa}}$$

$$\sigma_1 = 5,175 + 5,34$$

$$\boxed{\sigma_1 = 10,52 \text{ MPa}}$$

$$\sigma_2 = 5,175 - 5,34$$

$$\boxed{\sigma_2 = -0,165 \text{ MPa}}$$

$$\textcircled{6} \quad \sigma_x = 90 \text{ MPa}$$

$$\sigma_y = -80 \text{ MPa}$$

$$\tau_{xy} = 40 \text{ MPa}$$

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$$\sigma_{\text{med.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{med.}} = \frac{90 - 80}{2}$$

$$\sigma_{\text{med.}} = 5 \text{ MPa} \quad (\text{centro})$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{90 - (-80)}{2}\right)^2 + (40)^2}$$

$$R = 93,94 \text{ MPa} \quad * *$$

$$\sigma_1 = 5 + 93,94$$

$$\sigma_2 = 5 - 93,94$$

$$\sigma_1 = 98,94 \text{ MPa}$$

$$\sigma_2 = -88,94 \text{ MPa}$$

$$\sigma_{\text{med.}} = 100$$

$$\sigma_{\text{máx.}} = 98,94 \text{ MPa}$$

$$\sigma_{\text{int.}} = -88,94 \text{ MPa}$$

$$\sigma_{\text{mín.}} = -100 \text{ MPa}$$

$$\tau_{\text{máx. des.}} = \frac{\sigma_{\text{máx.}} - \sigma_{\text{mín.}}}{2}$$

$$\tau_{\text{máx. des.}} = \frac{98,94 - (-100)}{2}$$

$$\tau_{\text{máx. des.}} = 99,47 \text{ MPa} \quad * * *$$

$$\sigma_x = 0 \text{ MPa}$$

$$\sigma_y = 0 \text{ MPa}$$

$$\tau_{xy} = 5 \text{ MPa}$$

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$$\sigma_{\text{med.}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\text{med.}} = 0 \text{ MPa (centro)}$$

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{0-0}{2}\right)^2 + (5)^2}$$

$$r = 5 \text{ MPa} \quad \text{X X}$$

$$\sigma_1 = 0 + 5$$

$$\sigma_2 = 0 - 5$$

$$\sigma_1 = 5 \text{ MPa}$$

$$\sigma_2 = -5 \text{ MPa}$$

$$\sigma_{\text{máx}} = 7 \text{ MPa}$$

$$\sigma_{\text{int.}} = 5 \text{ MPa}$$

$$\sigma_{\text{mín}} = -5 \text{ MPa}$$

$$\tau_{\text{máx des}} = \frac{\sigma_{\text{máx}} - \sigma_{\text{mín}}}{2}$$

$$\tau_{\text{máx des}} = \frac{7 - (-5)}{2}$$

$$\tau_{\text{máx des}} = 6 \text{ MPa}$$

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⊗ $\sigma_x = 40 \text{ MPa}$

$\sigma_y = -40 \text{ MPa}$

$\tau_{xy} = 0 \text{ MPa}$

Plano X-Y

~~$\sigma_{\text{méd}} = 0 \text{ MPa}$~~

$\sigma_{\text{méd}} = \frac{\sigma_x + \sigma_y}{2}$

$\sigma_{\text{méd}} = \frac{40 - 40}{2}$

$\sigma_{\text{méd}} = 0 \text{ MPa}$

$\tau_{\text{máx}} = \frac{\sigma_{\text{máx}} - \sigma_{\text{mín}}}{2}$

$\tau_{\text{máx}} = \frac{40 - (-40)}{2}$

$\tau_{\text{máx}} = 40 \text{ MPa}$

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Plano Y-Z

$\sigma_y = -40 \text{ MPa}$

$\sigma_z = -40 \text{ MPa}$

$\tau_{yz} = 0 \text{ MPa}$

$\sigma_{\text{méd}} = \frac{\sigma_y + \sigma_z}{2}$

$\sigma_{\text{méd}} = \frac{-40 - 40}{2}$

$\sigma_{\text{méd}} = -40 \text{ MPa}$

$\tau_{\text{máx}} = \frac{\sigma_{\text{máx}} - \sigma_{\text{mín}}}{2}$

$\tau_{\text{máx}} = \frac{-40 - (-40)}{2}$

$\tau_{\text{máx}} = 0 \text{ MPa}$

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Plano X-Z

$\sigma_x = 40 \text{ MPa}$

$\sigma_z = -40 \text{ MPa}$

$\tau_{xz} = 0 \text{ MPa}$

$\sigma_{\text{méd}} = \frac{\sigma_x + \sigma_z}{2}$

$\sigma_{\text{méd}} = \frac{40 + (-40)}{2}$

$\sigma_{\text{méd}} = 0 \text{ MPa}$

$\tau_{\text{máx}} = \frac{\sigma_{\text{máx}} - \sigma_{\text{mín}}}{2}$

$\tau_{\text{máx}} = \frac{40 - (-40)}{2}$

$\tau_{\text{máx}} = 40 \text{ MPa}$

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